Rigid Body Rotation (cont'd)

Week 10, Lesson 1

- Parallel-Axis Theorem
- Combined Rotation & Translation
- Angular Momentum

References/Reading Preparation:
Schaum's Outline Ch. 10
Principles of Physics by Beuche – Ch.8

Summary From Last Lecture

1) An object of mass M possesses rotational inertia, where,

$$I = Mk^2$$

2) A rotating object has rotational kinetic energy, where,

$$KE_r = \frac{1}{2} I\omega^2$$

3) A torque (τ) applied to an object that is free to rotate gives the object an angular acceleration, where,

$$\tau = I\alpha$$

4) The work done by a torque, τ, when it acts through an angle θ is $\tau\theta$.

Parallel-Axis Theorem

The moments of inertia of the objects shown in your text are calculated about the centres of the mass of the objects.

There s a very simple and useful theorem by which we can calculate the moments of inertia of these same objects about *any other axis* which is parallel to the centre of mass axis.

The moment of inertia of an object about an axis O which is parallel to the centre of mass of the object is:

$$I = I_c + Mh^2$$

Where, I_c = moment of inertia about an axis through the mass centre

M = total mass of the body

h = perpendicular distance between the two parallel axes

Worked Example

Determine the moment of inertia of a solid disk of radius r and mass M about an axis running through a point on its rim and perpendicular to the plane of the disk.

(ans. 3/2 Mr^2)

Worked Example

Find the rotational kinetic energy of the earth due to its daily rotation on its axis. Assume a uniform sphere of M = 5.98 x 10^{24} kg, r = 6.37 x 10^{6} m

Worked Example

A certain wheel with a radius of 40 cm has a mass of 30 kg and a radius of gyration, *k*, of 25 cm. A cord wound around its rim supplies a tangential force of 1.8 N to the wheel which turns freely on its axis. Find the angular acceleration of the wheel.

(ans. $\alpha = 0.384 \text{ rad/s}^2$)

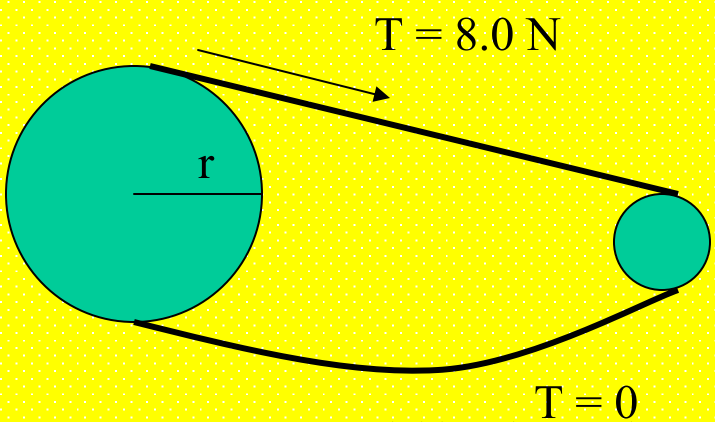
Worked Example

The larger wheel shown has a mass of 80 kg and a radius *r* of 25 cm. It is driven by a belt as shown. The tension in the upper part of the belt is 8.0 N and that for the lower part is essentially zero. Assume the wheel to be a uniform disk.

a) How long does it take for the belt to accelerate the larger wheel from rest to a speed of 2.0 rev/s?

b) How far does the wheel turn in this time (i.e., what is the angular displacement, θ)?

c) What is the rotational KE?



(ans. $t = 15.7$ s
$\theta = 98.6$ rad
$KE_r = 197$ J)

Worked Example

A 500 g uniform sphere of 7.0 cm radius spins at 30 rev/s on an axis through its centre. Find its:

a) KE_r (ans. 17.3 J)

b) Angular momentum (ans. 0.184 $kg \cdot m^2/s$)

c) Radius of gyration (ans. 0.0443 m)

Combined Rotation and Translation

The kinetic energy, KE, of a rolling ball or other rolling object of mass *M* is the sum of:

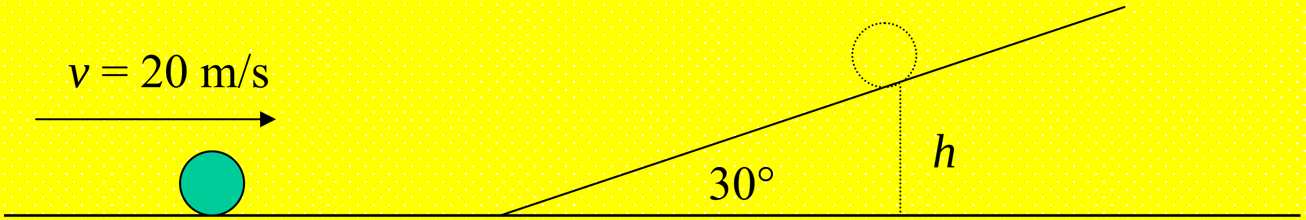
1) Its rotational KE *about an axis through its centre of mass*, and
2) The translational KE of an equivalent point mass moving with the centre of mass.

$$KE\ total = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

Note that I is the moment of inertia of the object about an axis through its mass centre.

Worked Example

As shown, a uniform sphere rolls on a horizontal surface at 20 m/s and then rolls up the incline. If friction losses are negligible, what will be the value of *h* where the ball stops?



(ans. h = 28.6 m)

Angular Momentum

Rotational, or angular, momentum is associated with the fact that a rotating object persists in rotating.

Angular momentum is a vector quantity with magnitude $I\omega$ and is directed along the axis of rotation.

If the net torque on a body is zero, its angular momentum will Remain unchanged in both magnitude and direction. This is the *Law of conservation of angular momentum.*

Worked Example

A disk of moment of inertia I_1 is rotating freely with angular speed ω_1 when a second, non-rotating, disk with moment of inertia I_2 is dropped on it. The two then rotate as a unit. Find the angular speed.

ans. $\omega = (I_1\omega_1)/(I_1 + I_2)$